# Sampling and Analysis for GNSS Radio Occultation

Stephen Leroy, Yi Huang

*Harvard University*

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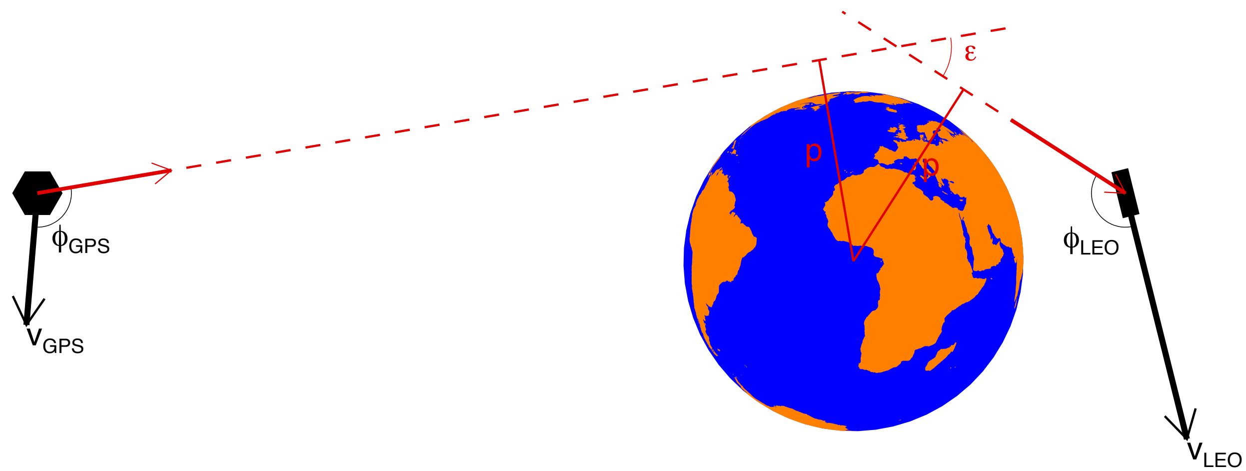
# Outline

- Radio occultation (RO) on CLARREO
  - *SI traceability*
  - *Information content*
- Accuracy requirements
  - *Systematic vs. random*
  - *Sounding vs. climatological*
- Sampling error
  - *Systematic sampling error*
  - *Analysis: Bayesian interpolation*
  - *Early results*

# GNSS Radio Occultation

$$-\frac{dL}{dt} = \lambda\,\Delta\nu = v_{\mathrm{GPS}}\cos\phi_{\mathrm{GPS}} + v_{\mathrm{LEO}}\cos\phi_{\mathrm{LEO}}$$



# GNSS Radio Occultation (2)

- Orbit determination and clock correction, GPS and LEO: *dL/dt*
- Diffraction (and multipath) inversion: *ε(p)*

$$U(p) = \frac{1}{4\pi}\iint_S \left\{ U \frac{\partial}{\partial n}\left(\frac{e^{iks}}{s}\right) - \frac{e^{iks}}{s}\frac{\partial U}{\partial n} \right\} dS$$

- Inversion for refractivity: *N(r)*

$$\ln n(p) = \frac{1}{\pi}\int_p^{\infty} \frac{\varepsilon(p')\,dp'}{\sqrt{p'^2 - p^2}}$$


Kursinski, E.R., G.A. Hajj, J.T. Schofield, R.P. Linfield, and K.R. Hardy, 1997: Observing Earth's atmosphere with radio occultation measurements using the Global Positioning System. *J. Geophys. Res.*, **102**, 23429-23465.

Hajj, G.A., E.R. Kursinski, W.I. Bertiger, L.J. Romans, and S.S. Leroy, 2002: A technical description of atmospheric sounding by GPS occultation. *J. Atmos. Solar-Terr. Phys.*, **64**, 451-469.

Gorbunov, M.E., H.H. Benzon, A.S. Jensen, M.S. Lohmann, and A.S. Nielsen, 2004: Comparative analysis of radio occultation processing approaches based on Fourier integral operators. *Radio Sci.*, **39**, doi:10.1029/2003RS002916.

# GNSS Radio Occultation (3)

- Refractivity

$$N = (n-1)\times 10^6 = (77.6\,\mathrm{K\,hPa^{-1}})\frac{p}{T} + (363\times 10^3\,\mathrm{K^2\,hPa^{-1}})\frac{p_\mathrm{w}}{T^2}$$

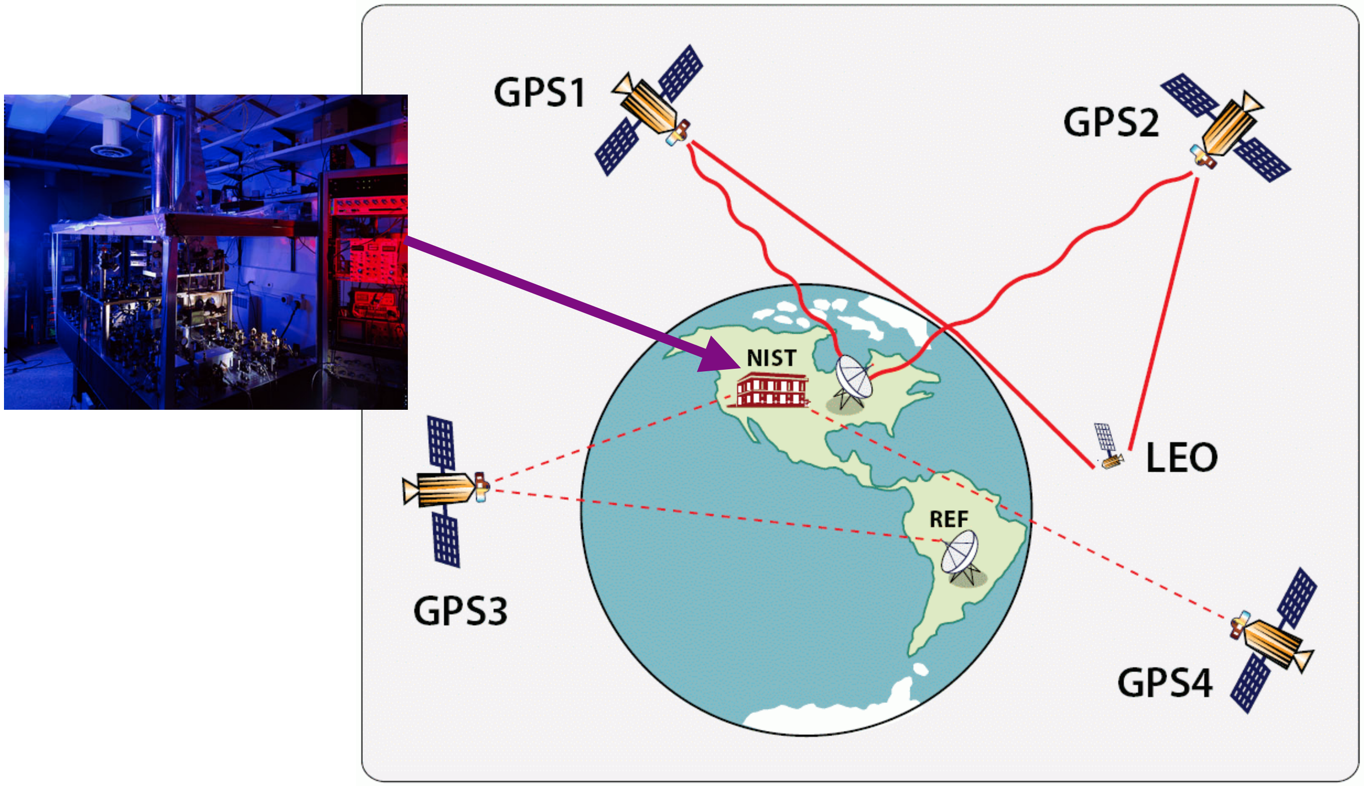
- "Dry" pressure

$$p_d(h) = (4.402\times 10^{-4}\,\mathrm{hPa\ m^{-1}})\int_h^\infty N\,dh \cong p(h) + (7521\,\mathrm{K})\int_0^{p(h)} \frac{q\,dp}{T}$$

- Geopotential height

$$h = \left[(\Phi(\mathbf{r}) - \frac{1}{2}\Omega^2 r_s^2) - (\Phi - \frac{1}{2}\Omega^2 r_s^2)_\mathrm{msl}\right] / g_0$$

# GNSS Radio Occultation Calibration




Hardy, K.R., G.A. Hajj, and E.R. Kursinski, 1994: Accuracies of atmospheric profiles obtained from GPS occultations. *Int. J. Sat. Comm.*, **12**, 463-473.

# Climate OSSEs

### Benchmark Measurement

- Traceable to international standards
- Minimize sampling error

### Climate OSSE

- Simulate trends in observable as produced by different models
- Explore information content with various contravariant fingerprints

### Climate Uncertainty

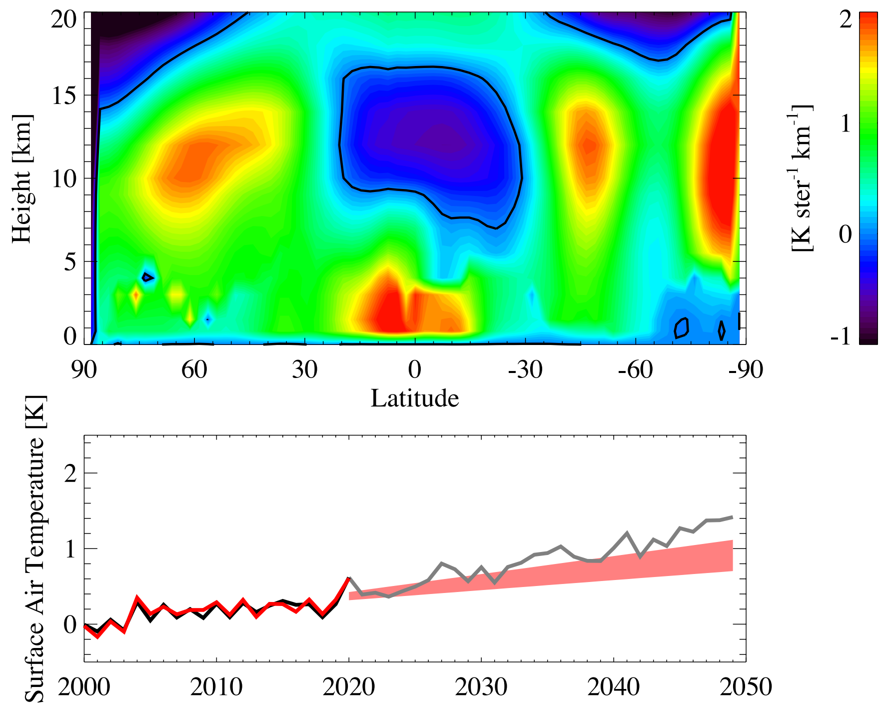
- Shortwave forcing
- Longwave forcing
- Climate feedbacks & sensitivity

### Climate OSSE Results

- Detection time and accuracy requirements
- How measurement constrains climate predictability
- Relative redundancy with other benchmark data types

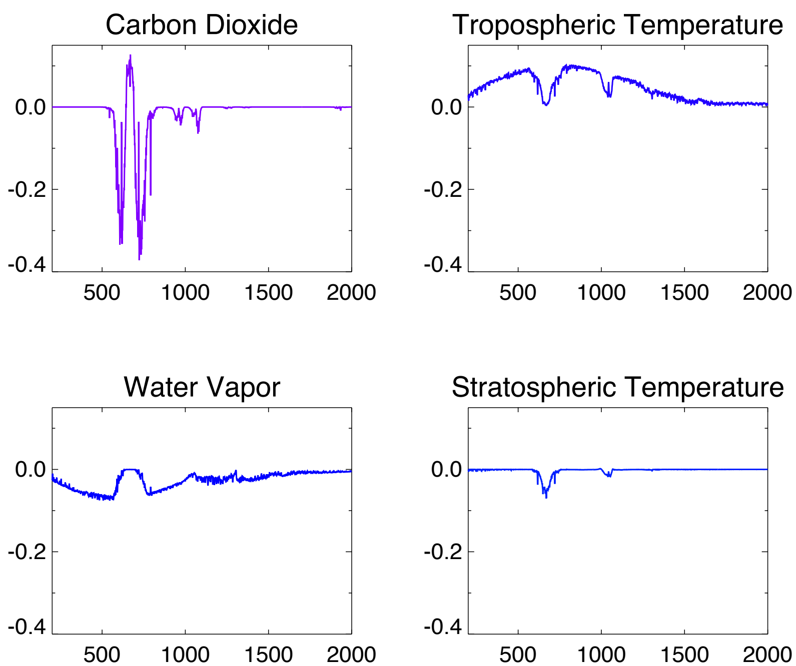
# CLARREO GNSS Radio Occultation

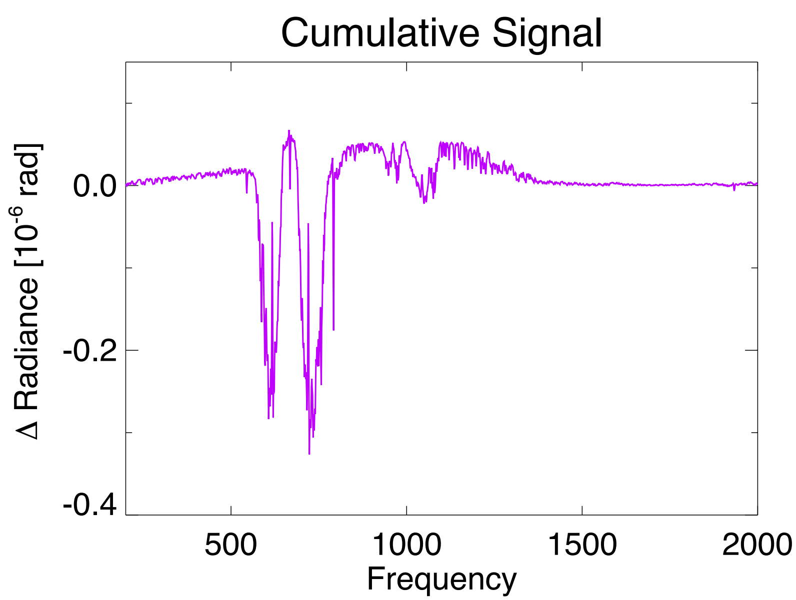
- A substitute for surface air temperature monitoring.




Leroy, Dykema, Gero, Anderson, 2009: Testing climate models using infrared spectra and GNSS radio occultation, in *New Horizons in Occultation Research*, Springer, 316pp.

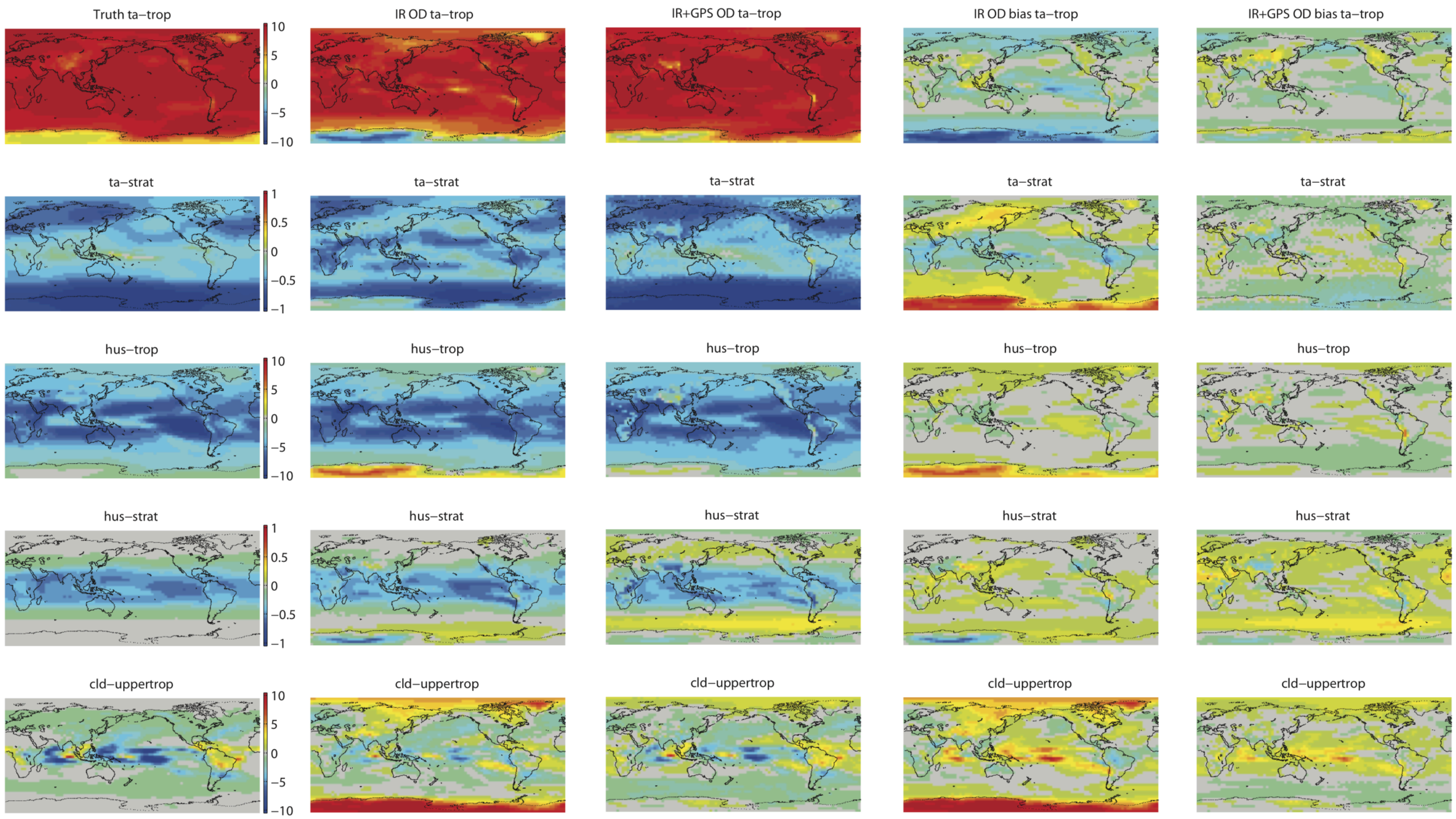
# Disentangling ambiguities in IR




Leroy, S.S., et al., 2008: Testing climate models using thermal infrared spectra. *J. Climate,* **21**, 1836-1848.

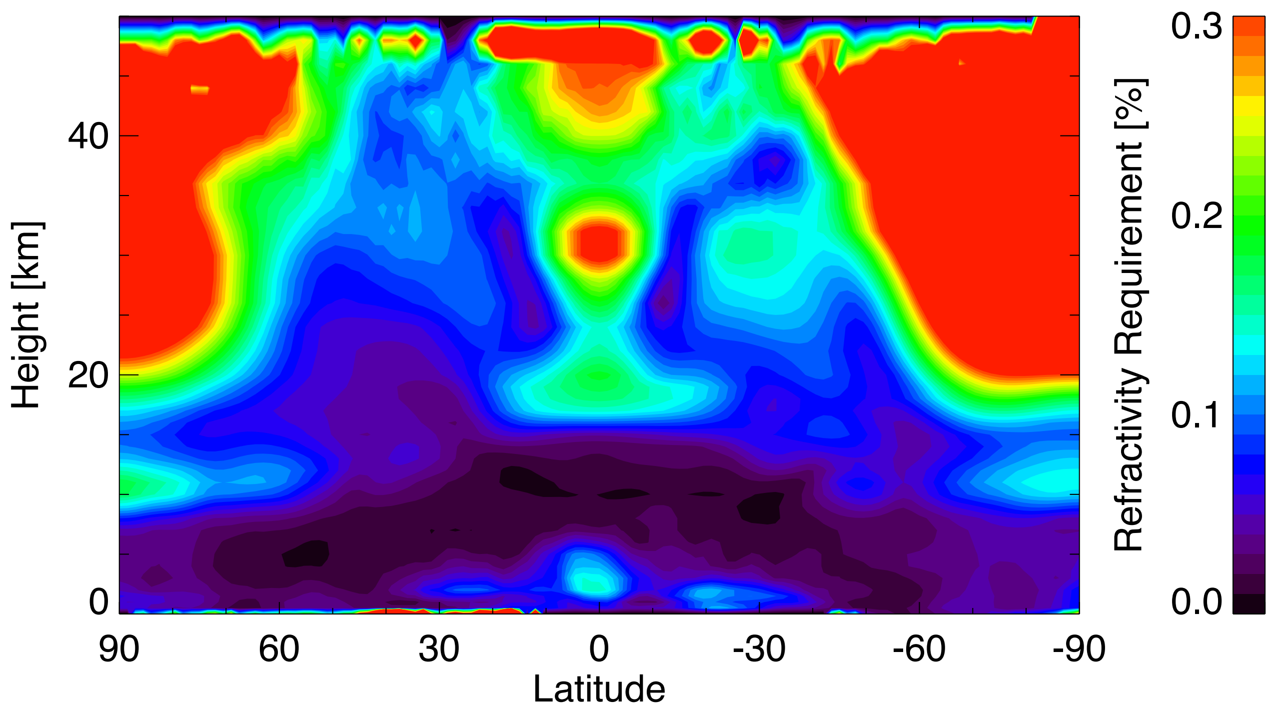
# Disentangling ambiguities in IR (2)




Huang et al., 2011: Determining longwave forcing and feedback using infrared spectra and GNSS radio occultation. *J. Climate,* In Press.

# GNSS RO Accuracy Requirements

Accuracy requirements are set wherein total uncertainty does not increase time-to-detection more than 10% above what is dictated by natural variability.




Leroy, Anderson, Ohring, 2008: Climate signal detection times and constraints on climate benchmark accuracy requirements. *J. Climate,* **21**, 841-846.

# Error Budget: GNSS RO

## Refractivity requirement: 0.03% at 18 km, 5° bins.

**Reviewed by the RO community, February 2, 2010.**

**CLARREO GNSS RO Error Budget**

| | **Troposphere (5-20km)** Testing that requirements are met for this region to be done at 18km. | | **Lower Troposphere (2-5km)** Testing that requirements are met for this region to be done at 3km. | |
|---|---|---|---|---|
| | **Phase rate error (mm/s)** | **Refractivity error (%)** | **Phase rate error (mm/s)** | **Refractivity error (%)** |
| **Measurement Requirement A: Individual Sounding** | | | | |
| **Systematic** | | | | |
| RO antenna phase center determination | 0.021 | 0.0007% | 0.021 | 0.0007% |
| Atmospheric multipath | 0.000 | | 0.000 | |
| Ionospheric residual | 0.300 | 0.0100% | 0.060 | 0.0020% |
| LEO POD | 0.054 | 0.0018% | 0.000 | 0.0000% |
| Clock accuracy | 0.030 | 0.0010% | 0.000 | 0.0000% |
| Nonlinearity (retrieval?) | | | 0.150 | 0.0050% |
| Attitude knowledge | 0.024 | 0.0008% | 0.024 | 0.0008% |
| Attitude rate knowledge | 0.150 | 0.0050% | 0.030 | 0.0010% |
| Local multi-path | 0.240 | 0.0080% | 0.060 | 0.0020% |
| *Total* | 0.418 | 0.0139% | 0.178 | 0.0059% |
| **Random** | | | | |
| Instrument precision | 0.240 | 0.0080% | 0.060 | 0.0020% |
| Ionospheric scintillation | 0.060 | 0.0020% | 0.000 | 0.0000% |
| Gravity waves | 0.090 | 0.0030% | 0.000 | 0.0000% |
| Clock precision | 0.099 | 0.0033% | 0.000 | 0.0000% |
| *Total* | 0.281 | 0.0094% | 0.060 | 0.0020% |
| **Measurement Requirement B: Climatological Averaging** | | | | |
| **Systematic** | | | | |
| Diurnal cycle | n/a | 0.0100% | n/a | 0.0300% |
| *Total* | n/a | 0.0100% | n/a | 0.0300% |
| **Random** | | | | |
| Sampling density | n/a | 0.0220% | n/a | 0.0800% |
| *Total* | n/a | 0.0220% | n/a | 0.0800% |
| *Total systematic error (target)* | | *0.0170%* | | *0.0570%* |
| **Total systematic error** | | **0.0172%** | | **0.0306%** |
| *Total random error (target)* | | *0.0250%* | | *0.0820%* |
| **Total random error** | | **0.0239%** | | **0.0800%** |

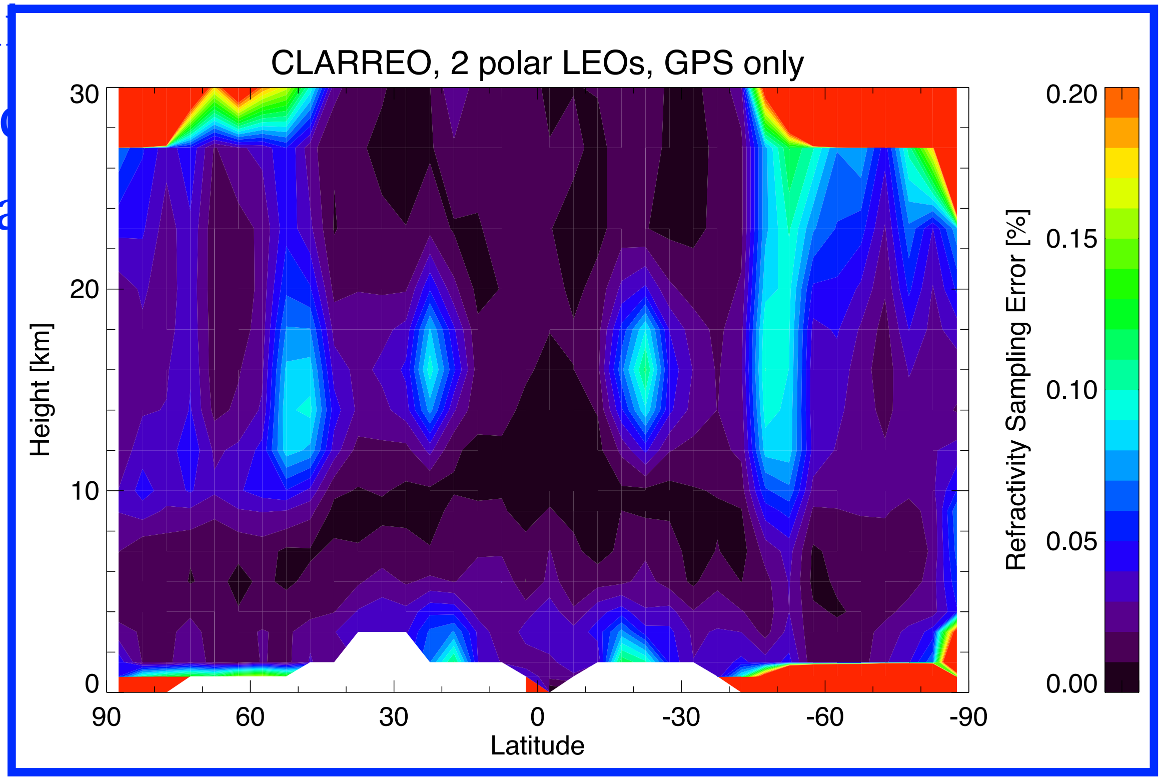
0.022%

# Sampling Error

- Simulate a distribution of soundings.
- Interpolate reanalysis to time and location of soundings.
- Form climatology based on reanalysis “data”.
- Compare to reanalysis gridded “truth”.

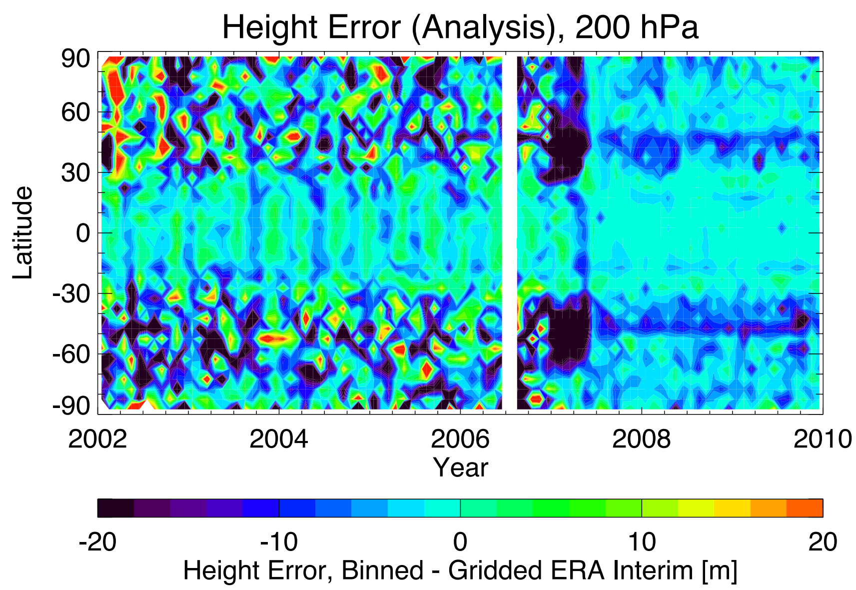
# Sampling Error

- Simulate a distribution of soundings.
- Interpolate reanalysis to time and location of soundi
- Form
- Compa



# Sampling Error, CHAMP & COSMIC

## Geopotential height of 200 hPa *dry* pressure surface



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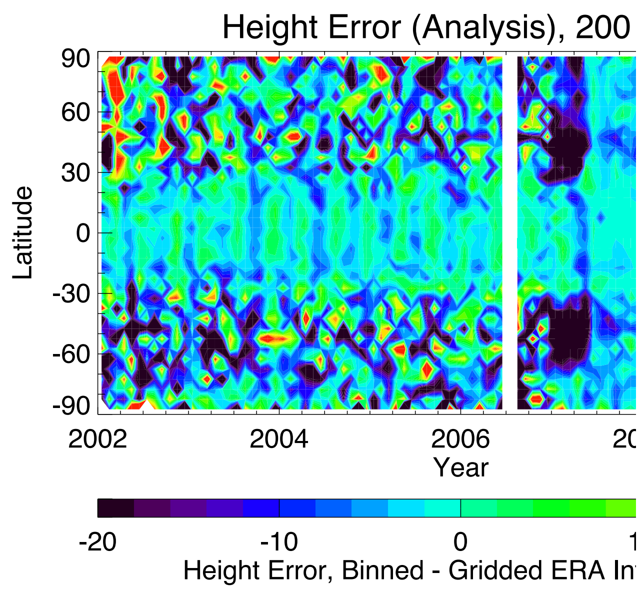


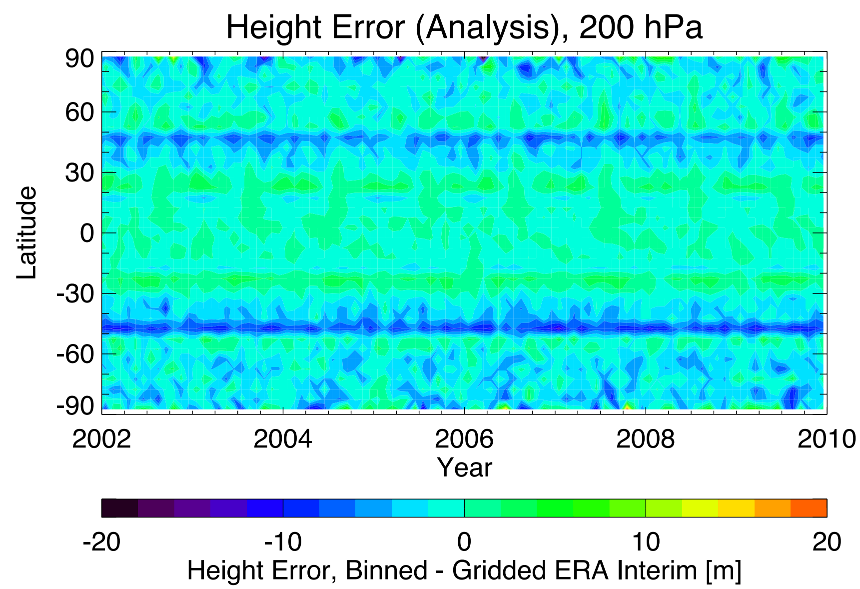

# Sampling Error, CHAMP & COSMIC

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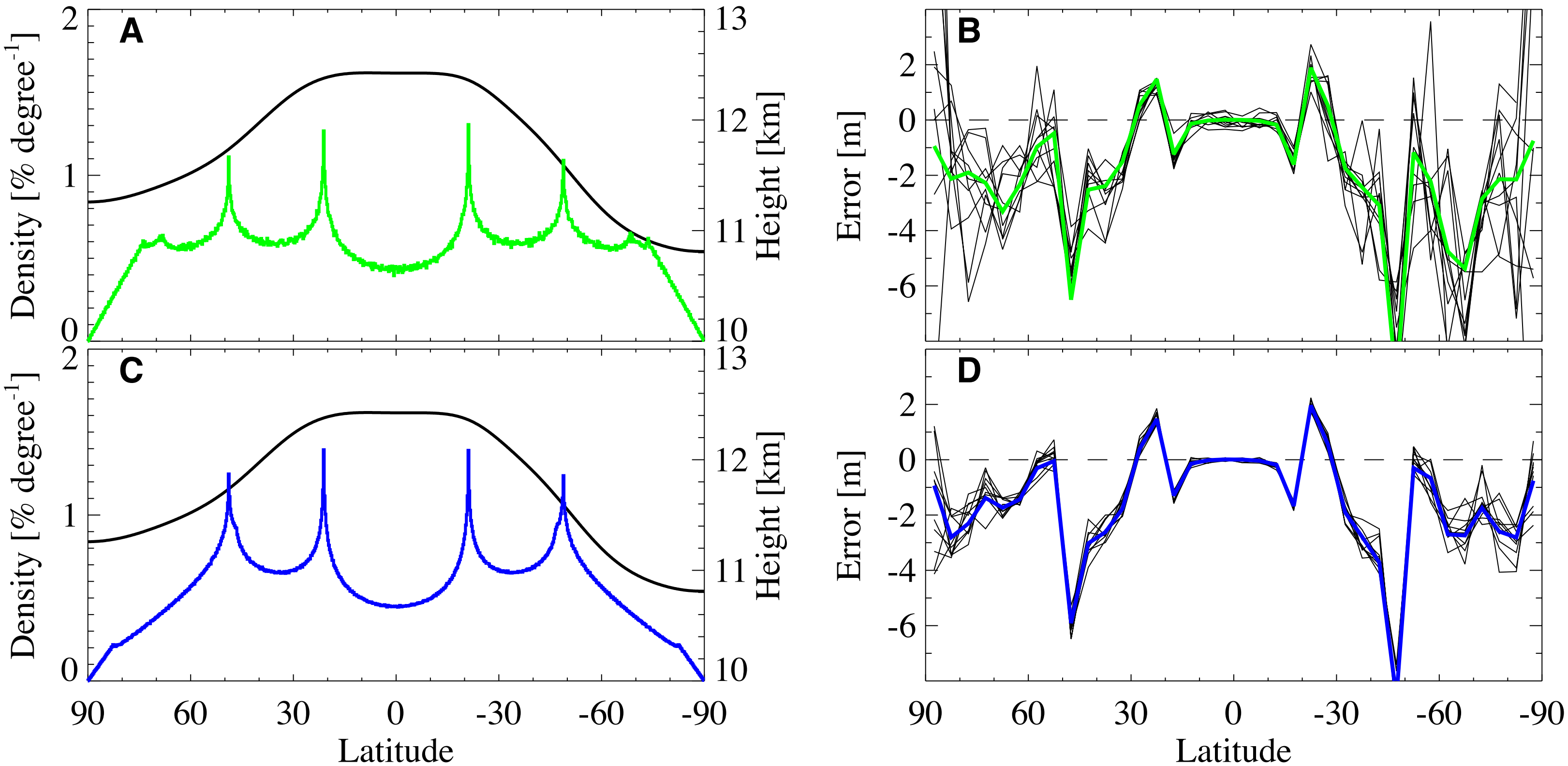
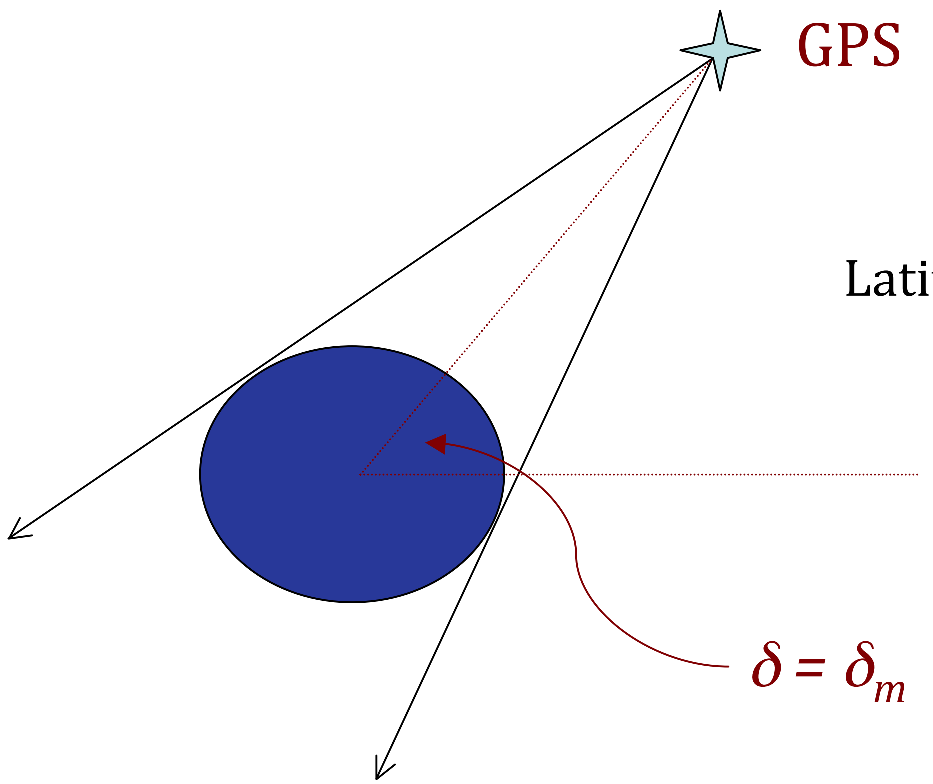


# Systematic sampling error cause




Latitude = 48.84°, 21.14° N & S

# Analysis: Bayesian Interpolation

**Challenge**: Fit data without under-fitting or overfitting. Data under-samples unknown synoptic variability in space and time.

**Solution**: Use spherical harmonics as basis functions, apply hierarchical Bayesian inference to find best fit and most likely weighting of data fitting and penalty function.

$$\chi^2 = \underbrace{\beta\left|\mathbf{t} - \phi\mathbf{w}\right|^2}_{\text{Fit}} + \underbrace{\alpha\,\mathbf{w}'\mathbf{C}\mathbf{w}}_{\text{Penalty}}$$

# Analysis: Bayesian Interpolation (2)

$$\chi^2 = \beta\left|\mathbf{t} - \phi\mathbf{w}\right|^2 + \alpha\,\mathbf{w}'\mathbf{C}\mathbf{w}$$

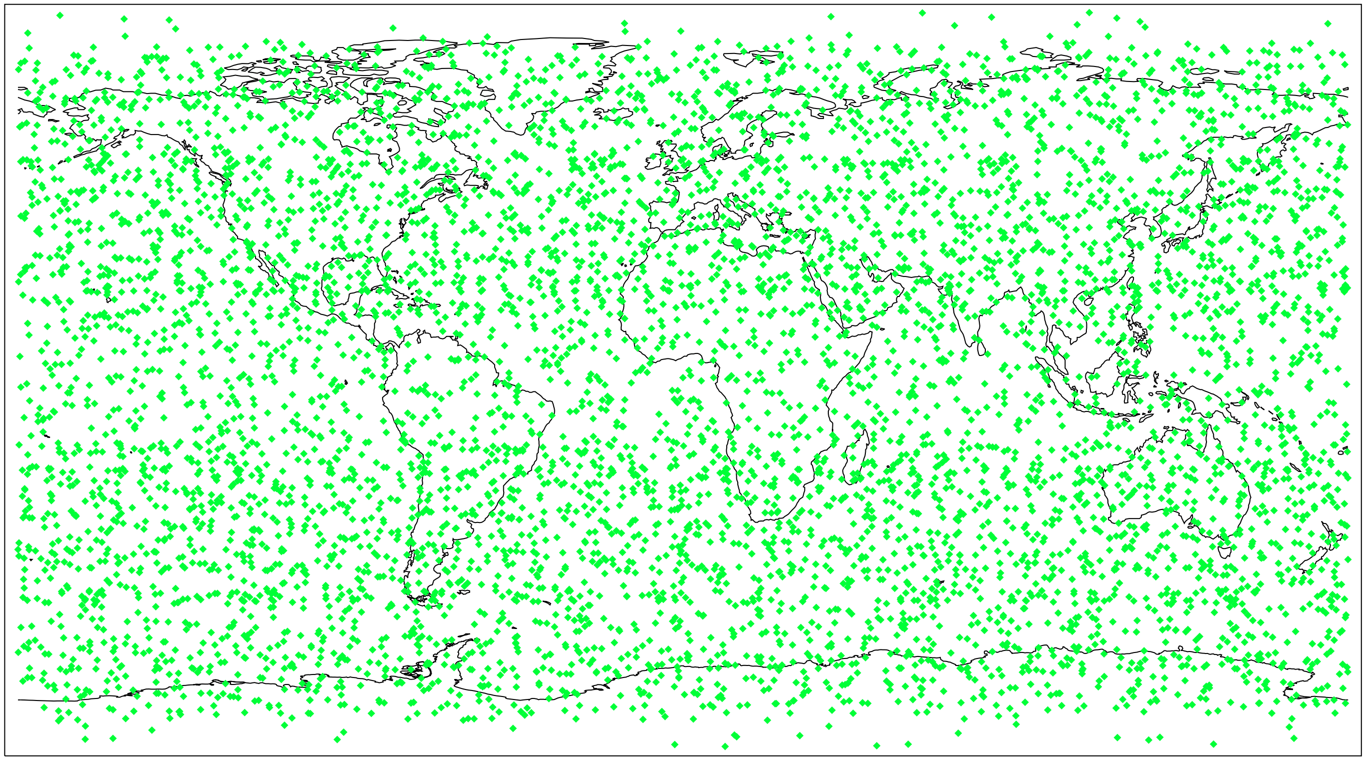
$$\mathbf{B} = \phi\phi'$$

$$\mathbf{A} = \beta\mathbf{B} + \alpha\mathbf{C}$$

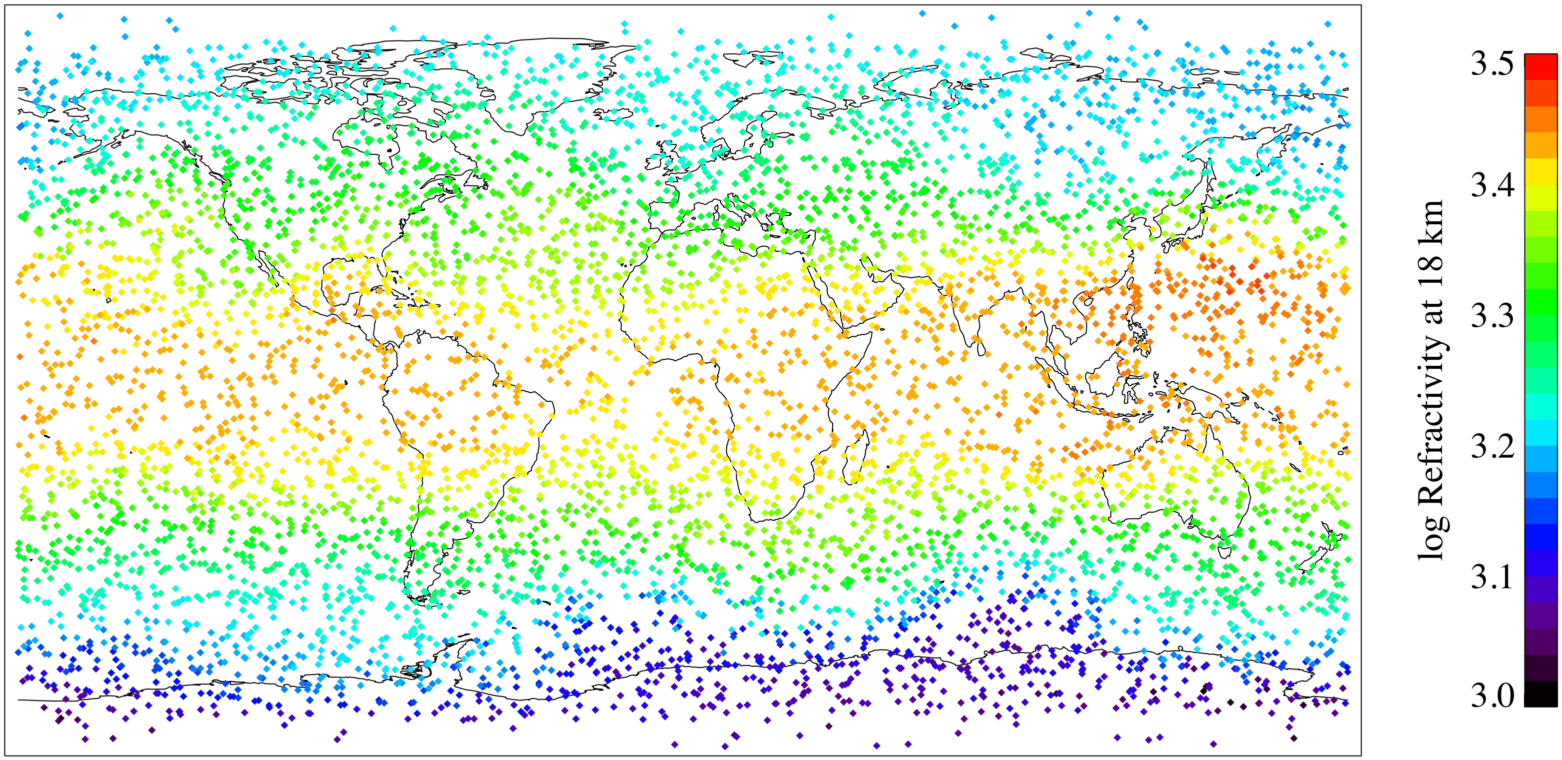
First inference: $\mathbf{w} = \mathbf{A}^{-1}\beta\phi'\mathbf{t}$

Second inference: $\gamma = k - \alpha\,\mathrm{Trace}\,\mathbf{A}^{-1}\mathbf{C} = N - \beta\left|\mathbf{t} - \phi\mathbf{w}\right|^2$

# Analysis: Bayesian Interpolation (3)



# Analysis: Bayesian Interpolation (3)



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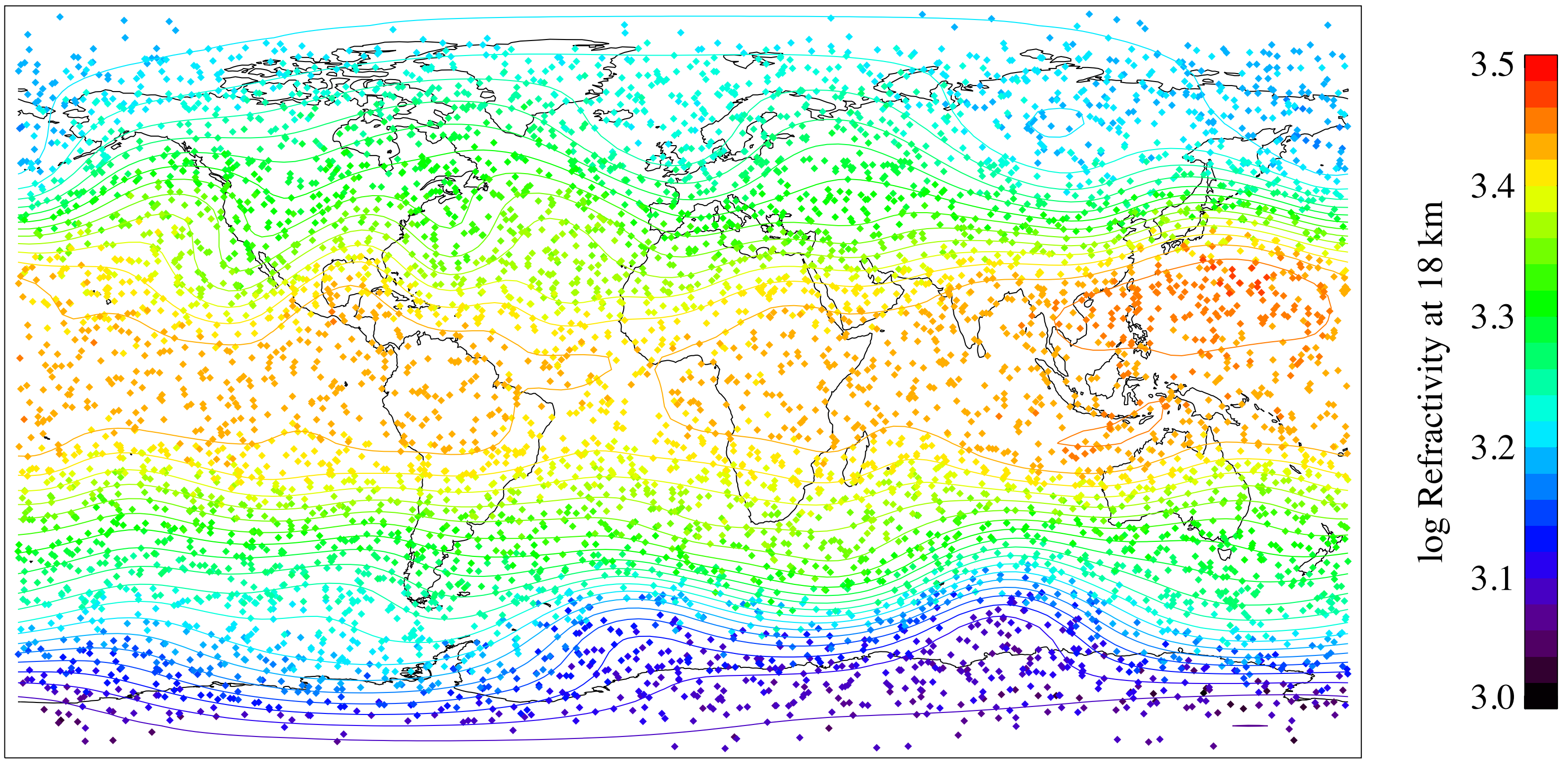
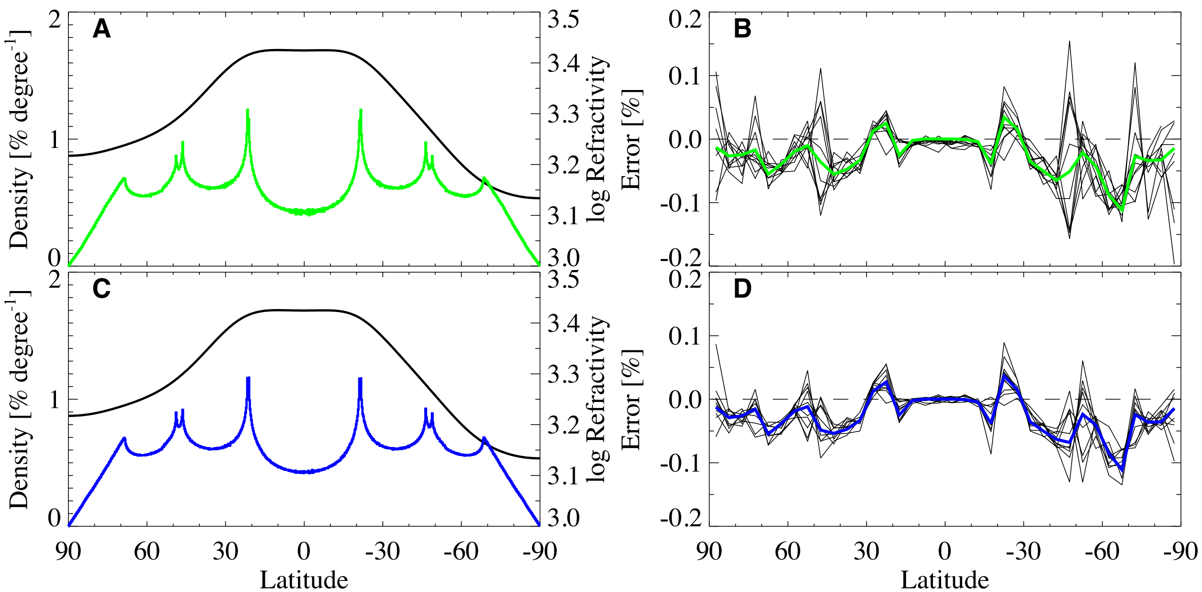
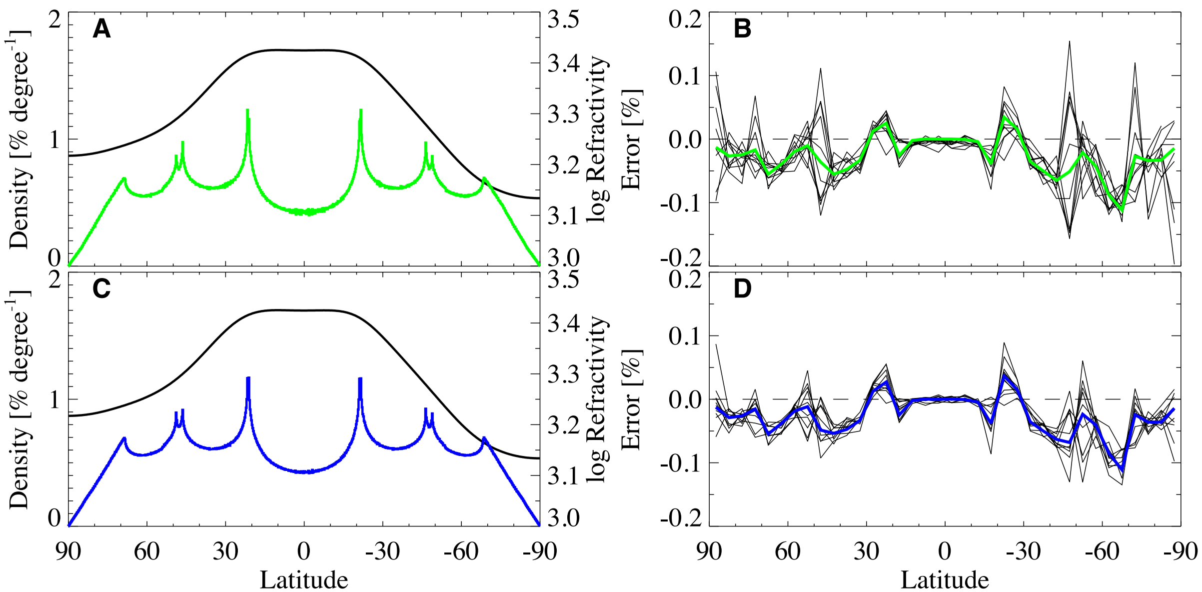


# Systematic Sampling Error

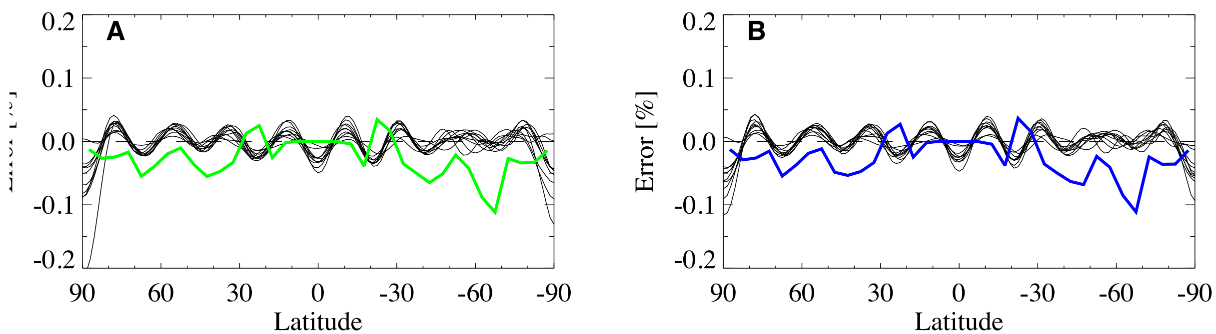



Binning and averaging

# Systematic Sampling Error




Binning and averaging




Bayesian interpolation

# Summary

- A systematic sampling error arises due to weak singularities in RO coverage. The singularities are at 48° and 21° north and south latitude.
- Bayesian interpolation greatly reduces systematic error and random error. Residual systematic error due to spherical harmonic truncation.
- CLARREO sampling requirements will clearly be effected, most likely downward. Yet to determine temporal sampling errors and consequently the number of required CLARREO orbits.

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Extra slides

# GNSS RO Missions

- **GPS/MET**, UCAR, 1995-1997. Soundings in four "prime time" periods.
- **CHAMP**, GFZ Potsdam, 2001-2009. ~200 soundings daily.
- **MetOp-A**, EUMETSAT, 2006-present. ~500 soundings daily; intermittent availability.
- **COSMIC**, UCAR, 2006-present. ~2800 soundings daily; degrading because of age.
- **TerraSAR-X**, DLR, 2008-present. Presently unavailable.

# More general: Bayes (or just marginalization)

$$P(\mathbf{x},\mathbf{y}) = P(\mathbf{x} \mid \mathbf{y})\, P(\mathbf{y}) = P(\mathbf{y} \mid \mathbf{x})\, P(\mathbf{x})$$

$\mathbf{x}$ : observable variables

$\mathbf{y}$ : prediction variables

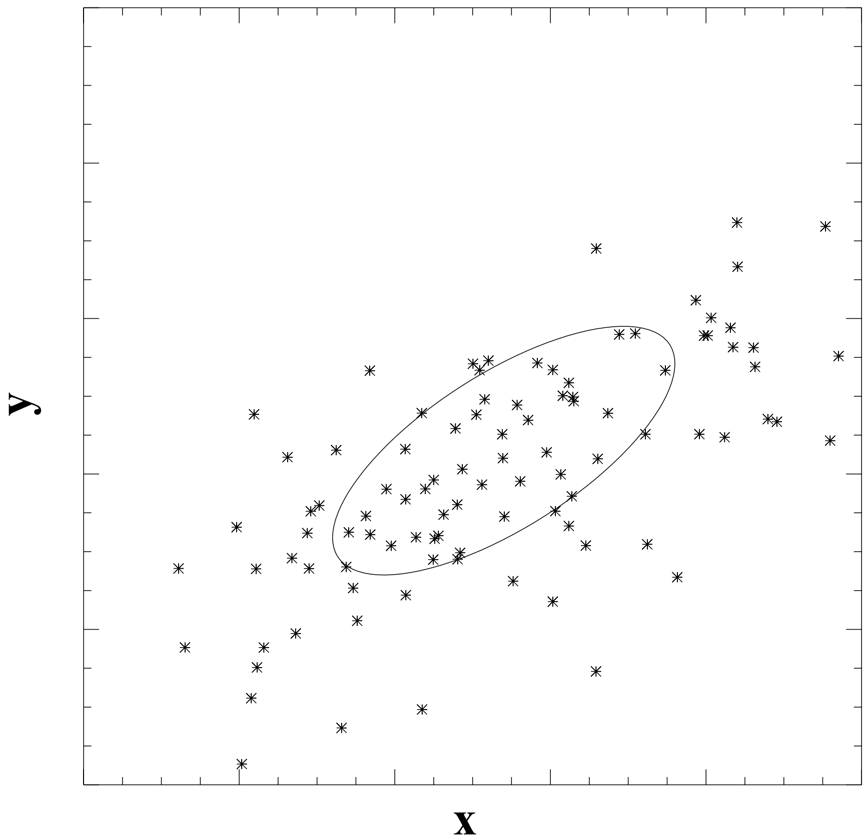
Form joint PDF $P(\mathbf{x},\mathbf{y})$ using an ensemble of climate models. For each model, need (1) observation kernel to simulate data $\mathbf{x}$ from hindcast run, and (2) emissions scenario run to generate prediction variables $\mathbf{y}$.

Natural variability in $\mathbf{x}$ and $\mathbf{y}$ and uncertain physics will both be accounted for.

With data $\mathbf{d}$, set $\mathbf{x} = \mathbf{d}$ and $P(\mathbf{y}|\mathbf{x}=\mathbf{d})$ is the projection PDF with data incorporated. $P(\mathbf{x})$ is a normalization constant that guarantees a unit integral of $P(\mathbf{y}|\mathbf{x})$ over $\mathbf{y}$.

# Simple Solution

Assume Gaussian statistics




Covariance for $(x,y)$: $\Sigma = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$

Projection $y$: $\bar{y}_{\text{posterior}} = \bar{y}_{\text{prediction}} + \rho \dfrac{\sigma_y}{\sigma_x} d$

Projection uncertainty: $\sigma_{y,\text{posterior}}^2 = \sigma_y^2 \, (1 - \rho^2)$

# Multivariate Solution

$$\Sigma = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$$

Correlation of prediction and data

Optimization

$$\mathbf{y}_{\text{posterior}} = \bar{\mathbf{y}}_{\text{prior}} + \Sigma_{yx}\Sigma_{xx}^{-1}(\mathbf{d} - \bar{\mathbf{x}}_{\text{prior}})$$

$$\Sigma_{y|d} = \Sigma_{yy} - \underbrace{\Sigma_{yx}\Sigma_{xx}^{-1}\Sigma_{xy}}_{\text{Information provided by data}}$$

Ensemble uncertainty

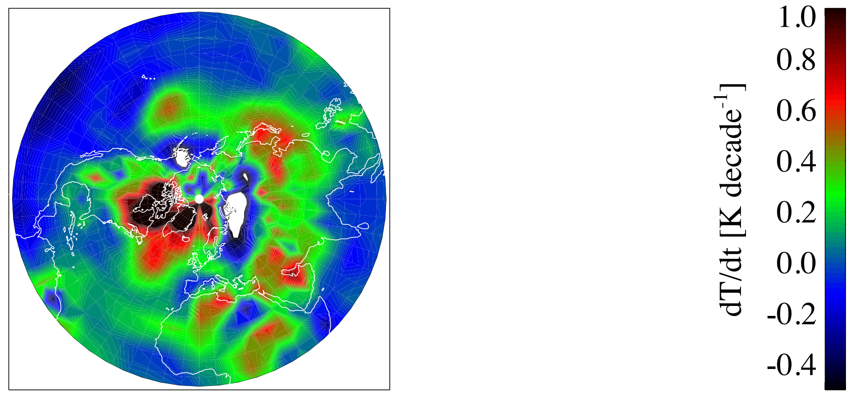
Information provided by data

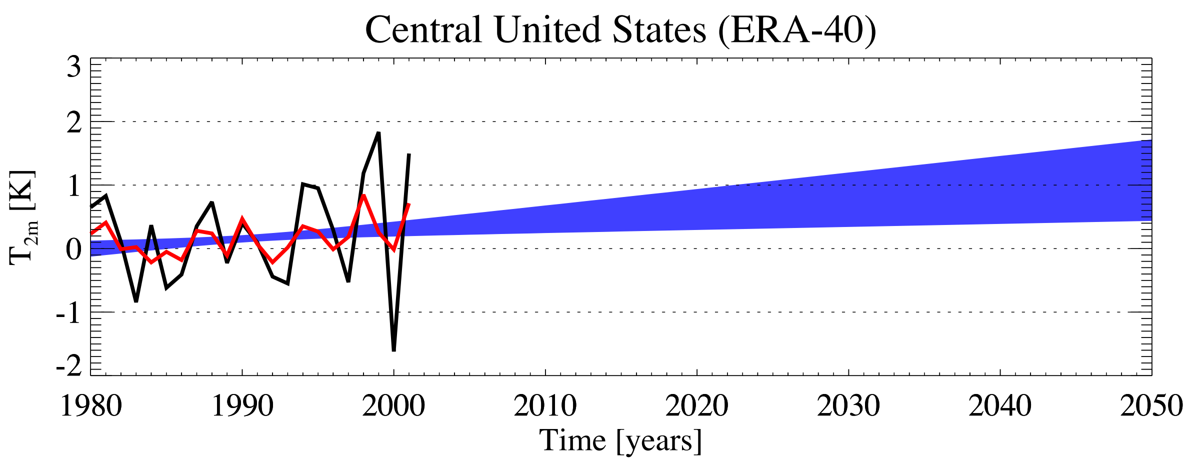
- Continuum of models is assumed, solving the problem of reality ensemble averaging (REA)
- Higher dimensionality in x (observation) reduces posterior uncertainty but requires greater number of models in ensemble.
- Information is provided by correlation. *Which data types are the most important for improving climate prediction?*

# Information in data types

Infer 50-yr temperature trend from observed 20-yr trends:

# Application of Reanalyses: Central U.S.



# Application of Reanalyses: Sahel